B.Sc Part I
Paper I

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3) Relativity of Space : Lorentz-Fitzgerald Contraction (or Length Contraction) :

Consider two systems S and S', the latter moving with velocity v relative to former along +ve direction of X-axis.

Consider a rod parallel to x-axis placed in system S' at rest. Let the x-co-ordinate of the ends of the rod in system S' be at x_1' and x_2'. Then the length of the rod in system $S' = (x_2' - x_1') = L_0$ (Say)

i.e., $L_0 = x_2' - x_1'$.

Let there be an observer in system S. obviously the observer O in S is moving with velocity v relative to the rod. Now we have to determine the length of the rod from system S.

Let x_1 and x_2 be the Coordinate along x-axis of the ends of the rod at time t, t being the same for the both ends i.e., the observations of both ends of rod are taken simultaneously.

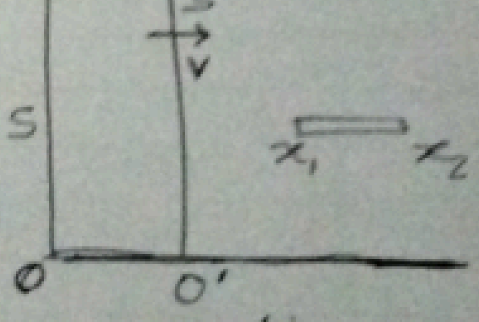

fig-1

Thus the length of the rod (at rest in S) observed by an observer from system $S = L = (x_2 - x_1)$.
Now from Lorentz inverse transformation equation we have,

$$x = \frac{x' + vt'}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}} \quad \text{where } t' = \frac{t - (vx/c^2)}{\sqrt{(1 - v^2/c^2)}}$$

$$\therefore \quad x = \frac{x' + v\left\{\left(t - \frac{vx}{c^2}\right) \Big/ \sqrt{\left(1 - \frac{v^2}{c^2}\right)}\right\}}{\sqrt{\left(1 - \frac{v^2}{c^2}\right)}}$$

$$= \frac{x'}{\sqrt{(1 - v^2/c^2)}} + \frac{vt - v^2x/c^2}{1 - v^2/c^2}$$

$$\therefore \left(1 + \frac{v^2/c^2}{(1 - v^2/c^2)}\right) x = \frac{x'}{\sqrt{(1 - v^2/c^2}} + \frac{vt}{1 - v^2/c^2}$$

$$\therefore \quad \frac{x}{1 - v^2/c^2} = \frac{x'}{\sqrt{(1 - v^2/c^2}} + \frac{vt}{(1 - v^2/c^2)}$$

$$\therefore \quad x = x' \sqrt{\{1 - v^2/c^2\}} + vt$$

As both ends of the rod are being observed at same instant t, we have

$$x_1 = x_1' \sqrt{\left(1 - \frac{v^2}{c^2}\right)} + vt$$

$$x_2 = x_2' \sqrt{\left(1 - \frac{v^2}{c^2}\right)} + vt$$

Therefore, $x_2 - x_1 = (x_2' - x_1') \sqrt{(1 - v^2/c^2)}$

or $\quad L = L_0 \sqrt{\left(1 - \frac{v^2}{c^2}\right)}$

This shows that $L < L_0$.

This follows that the length of rod seems to be contracted when measured by moving observer along the direction of length of rod in the ratio $\left[1 : \left(1 - \frac{v^2}{c^2}\right)\right]$, v being the velocity of the observer.

If we consider the length of rod along perpendicular to direction of motion of frame (say along x-axis) and if y_1, y_2 are coordinates of ends of rod relative to system S corresponding to coordinates y_1' and y_2' in frame S', Then

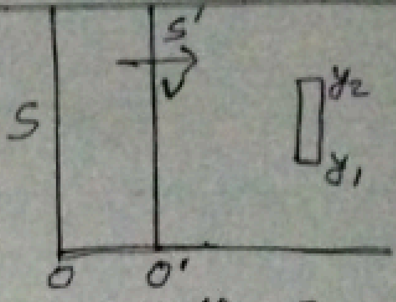

fig-2

Rest length of rod $L_{oy} = y_2' - y_1'$ as rod is at rest in system S'.

Length of rod in system S, $L_y = y_2 - y_1$

$\therefore$ From Lorentz transformation $y = y'$, we have

$$y_1 = y_1' \quad y_2 = y_2'$$

$$\therefore \quad y_2 - y_1 = y_2' - y_1'$$

i.e.,
$$\boxed{L_y = L_{oy}}$$

That is the length of rod remains unchanged in a direction perpendicular to the direction of motion.

Thus we conclude that

The length of moving rod is contracted along the direction of motion by a factor $\sqrt{(1-v^2/c^2)}$; while there is no contraction along a direction perpendicular to the direction of motion.

This statement is called Lorentz-Fitzgerald contraction.

Conclusively, " Every rigid body appears to have maximum dimensions when at relative to the observer.

Its dimensions appear to be contracted in the direction of relative motion by the factor $\sqrt{(1-v^2/c^2)}$ when it moves with velocity v relative to the observer."